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Time evolution of a time-dependent harmonic oscillator in a static magnetic field

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Abstract

The analytic expression of the time evolution wavefunction of the two-dimensional harmonic oscillator with time-dependent mass and frequency in a static magnetic field is obtained using an operator-algebraic method slightly different from the usual Lie algebraic technique. The evolution operator of the one-dimensional harmonic oscillator with time-dependent mass and frequency is established first by forming an operator differential equation with the $su(1,1)$ Lie algebra, which is deduced from the time-dependent linear unitary transformation for boson operators $(a, a^{\dagger})$, and then by comparing this operator equation with the time evolution equation of the one-dimensional oscillator.

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In a recent paper, Maamache *et al* [1] solved the time-dependent Schrödinger equation for the two-dimensional harmonic oscillator with time-dependent mass and frequency in the presence of a static magnetic field. The authors made use of the Lewis and Riesenfeld (LR) [2] invariant theory to find the wavefunction of the system, and revealed that the different steps adopted in [3] to get the final results are not correct. Because of the intrinsic mathematical interest and various applications to many areas of physics there are several other techniques to treat this problem besides the LR invariant method, such as the canonical transformation method [4], the Lie algebraic technique [5, 6], the path integral approach [7, 8] and the direct quantum mechanical treatment [9]. In this paper we shall take an alternative operator-algebraic approach different from the usual Lie algebraic method to study the same system as in [1] and [3]. Evaluation of the time evolution wavefunction of the system results essentially in performing calculations of the matrix elements for the rotation operator about the z-axial direction and for the evolution operator of the one-dimensional harmonic oscillator with time-dependent mass and frequency. The former matrix element is obtained by virtue of the decomposition method of the $su(2)$ Lie algebra, while for the latter its time evolution operator is established first by forming an operator differential equation with the $su(1,1)$ Lie algebra, which is deduced from the time-dependent linear unitary transformation for boson operators $(a, a^\dagger)$ [10], and then by comparing this operator equation with the time evolution equation of the one-dimensional oscillator.

The Hamiltonian we consider here is of the form [1, 3]

$$H(t) = \sum_{j=1}^{2} \mathcal{H}_j(t) + \frac{1}{2}\omega_c(t) L_z, \tag{1}$$

where

$$\mathcal{H}_j(t) = \frac{p_j^2}{2M(t)} + \frac{1}{2} M(t)\Omega^2(t) q_j^2 \tag{2}$$

is the Hamiltonian of a x- or y-component oscillator with time-dependent mass and frequency. The electromagnetic potential $\mathbf{A} = [-(B_0/2)q_2, (B_0/2)q_1, 0]$ is obtained in a choice of the Coulomb gauge $\nabla.\mathbf{A} = 0$; $\omega(t)$, $\omega_c(t) = eB_0/M(t)c$ and $\Omega^2(t) = \omega^2(t) + \omega_c^2(t)/4$ are the oscillating, Larmor and modulate frequencies, respectively; and L_z is the angular momentum in the axial direction.

It is known that $\sum \mathcal{H}_j(t)$ commutes with L_z [3]; this will facilitate the treatment of the time evolution equation $\mathrm{i}\hbar \dot{U}(t) = H(t)U(t)$ of the system (1). In terms of the Lie algebraic structure theory [11, 12] if we factor the evolution operator as the product $U(t) = R_z(t)\mathcal{U}_1(t)\mathcal{U}_2(t)$, the time evolution equation just mentioned will be decomposed into the following three equations:

$$\mathrm{i}\hbar \dot{R}_z(t) = \tfrac{1}{2}\omega_c(t) L_z R_z(t), \tag{3}$$

$$\mathrm{i}h\dot{\mathcal{U}}_j(t) = \mathcal{H}_j(t)\mathcal{U}_j(t), \qquad (j = 1, 2) \tag{4}$$

subject to the initial conditions $R_z(0) = 1$ and $\mathcal{U}_j(0) = 1$, where the dot means the time derivative. From equation (3), it follows that $R_z(t)$, which is called a rotation operator, is

$$R_z(t) = \exp\left[-\frac{\mathrm{i}}{\hbar}\xi_c(t) L_z\right], \tag{5}$$

where $\xi_c(t) = \int_0^t \mathrm{d}t' \omega_c(t')/2$ and $L_z = q_1 p_2 - q_2 p_1 = -\mathrm{i}\hbar\left(a_1^\dagger a_2 - a_1 a_2^\dagger\right)$. It is apparent that equation (4) presents the time evolution equation of the subsystem (2), while $\mathcal{U}_j(t)$ plays the role of a time evolution operator to the time-dependent harmonic oscillator with mass $M(t)$ and frequency $\Omega(t)$. For the moment, we ignore the subscript of $\mathcal{U}_j(t)$ since both the x- and y-component oscillators are equivalent. Equation (4) in the particle number representation may then be written as

$$\dot{\mathcal{U}}(t)\mathcal{U}^\dagger(t) = -\frac{\mathrm{i}}{\hbar}[\eta(t)(K_+ + K_-) + \kappa(t) K_0], \tag{6}$$

where

$$K_+ = \tfrac{1}{2}(a^\dagger)^2, \qquad K_- = \tfrac{1}{2}a^2, \qquad K_0 = \tfrac{1}{4}(aa^\dagger + a^\dagger a) \tag{7}$$

construct a $su(1,1)$ Lie algebra with the closed commutation relations $[K_+, K_-] = -2K_0$, $[K_0, K_\pm] = \pm K_\pm$, and the functions $\eta(t)$ and $\kappa(t)$ are defined by

$$\eta(t) = \frac{\hbar}{2}\left[\frac{M(t)\Omega^2(t)}{m\omega_0} - \frac{m\omega_0}{M(t)}\right], \qquad \kappa(t) = \hbar\left[\frac{M(t)\Omega^2(t)}{m\omega_0} + \frac{m\omega_0}{M(t)}\right]. \tag{8}$$

Consider the time-dependent linear unitary transformation for boson operators $(a, a^\dagger)$ [10]

$$U'(t)aU'^\dagger(t) = u(t)a + v(t)a^\dagger,$$
$$U'(t)a^\dagger U'^\dagger(t) = u^*(t)a^\dagger + v^*(t)a, \tag{9}$$

where $U'(t)$ is the time-dependent unitary operator and $u(t)$ and $v(t)$ are the complex transformation coefficients which satisfy the canonical condition

$$|u(t)|^2 - |v(t)|^2 = 1, \tag{10}$$

and the creation and annihilation operators $a^\dagger$ and a are time independent in the Schrödinger picture. Now differentiating equation (9) with respect to time and using equation (9) and the unitarity $U'^\dagger U' = U'U'^\dagger = 1$, $U'\dot{U}'^\dagger = -\dot{U}'U'^\dagger$, we obtain

$$u[\dot{U}'U'^\dagger, a] + v[\dot{U}'U'^\dagger, a^\dagger] = \dot{u}a + \dot{v}a^\dagger,$$
$$u^*[\dot{U}'U'^\dagger, a^\dagger] + v^*[\dot{U}'U'^\dagger, a] = \dot{u}^*a^\dagger + \dot{v}^*a. \tag{11}$$

Multiplying equation (11) by u, v properly and their conjugate complex functions and using condition (10), we have

$$[\dot{U}'U'^\dagger, a] = g(t)a - f(t)a^\dagger,$$
$$[\dot{U}'U'^\dagger, a^\dagger] = g^*(t)a^\dagger - f^*(t)a, \tag{12}$$

where the functions $f(t)$ and $g(t)$ are defined by

$$f(t) = v(t)\dot{u}^*(t) - u^*(t)\dot{v}(t), \qquad g(t) = u^*(t)\dot{u}(t) - v(t)\dot{v}^*(t) = -g^*(t). \tag{13}$$

Again, properly multiplying equation (12) by the operators $a^\dagger$, a and using the commutation relation $[a, a^\dagger] = 1$, we obtain

$$\dot{U}'U'^\dagger - a\dot{U}'U'^\dagger a^\dagger + a^\dagger\dot{U}'U'^\dagger a = -f(a^\dagger)^2 + f^*a^2 + g(aa^\dagger + a^\dagger a). \tag{14}$$

It can be checked that the product operator $\dot{U}'U'^\dagger$ that satisfies the above equation should take the form

$$\dot{U}'(t)U'^\dagger(t) = f(t)K_+ - f^*(t)K_- - 2g(t)K_0. \tag{15}$$

In order to solve this operator equation, introducing the usual 2×2 matrix representation of $su(1,1)$,

$$K_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \qquad K_- = \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}, \qquad K_0 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{16}$$

substituting them into equation (15) and adopting the decomposition technique proposed by Fisher *et al* [13], we find

$$\begin{aligned}\dot{U}'(t)U'^\dagger(t) &= \begin{pmatrix} g^* & f \\ f^* & g \end{pmatrix} = \begin{pmatrix} \dot{u}^* & -\dot{v} \\ -\dot{v}^* & \dot{u} \end{pmatrix}\begin{pmatrix} u & v \\ v^* & u^* \end{pmatrix} \\ &= \frac{\partial}{\partial t}\left[\begin{pmatrix} 1 & -v/u \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1/u & 0 \\ 0 & u \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -v^*/u & 1 \end{pmatrix}\right] \\ &\quad \times \left[\begin{pmatrix} 1 & 0 \\ v^*/u & 1 \end{pmatrix}\begin{pmatrix} u & 0 \\ 0 & 1/u \end{pmatrix}\begin{pmatrix} 1 & v/u \\ 0 & 1 \end{pmatrix}\right].\end{aligned} \tag{17}$$

Thus a comparison between both the extreme sides of equation (17) gives the normal-order and antinormal-order expansions of $U'(t)$ and $U'^\dagger(t)$, respectively, by

$$U'(t) = \exp\left[-\frac{v(t)}{u(t)}K_+\right]\exp[-2\ln u(t)K_0]\exp\left[\frac{v^*(t)}{u(t)}K_-\right], \tag{18}$$

$$U'^{\dagger}(t) = \exp\left[-\frac{v^*(t)}{u(t)}K_-\right]\exp[2\ln u(t)K_0]\exp\left[\frac{v(t)}{u(t)}K_+\right]. \tag{19}$$

On account of the same algebraic structure and physical meaning of equations (15) and (6), $U'(t)$ may be regarded as identical with $\mathcal{U}(t)$. Note that the time evolution operators $\mathcal{U}_1(t)$ and $\mathcal{U}_2(t)$ should obviously possess the completely same coefficients $u(t)$ and $v(t)$ except that their generators $(K_{\pm,0})$ are expressed in terms of boson operators $a_1\left(a_1^{\dagger}\right)$ and $a_2\left(a_2^{\dagger}\right)$ of different modes.

Concerning the connection between the transformation coefficients $u(t)$, $v(t)$ and the oscillator parameters $M(t)$, $\Omega(t)$, let us equate the right-hand sides of equations (15) and (6), which leads to the relations

$$f(t) = -\frac{\mathrm{i}}{\hbar}\eta(t), \qquad g(t) = \frac{\mathrm{i}}{2\hbar}\kappa(t). \tag{20}$$

Further using equation (13), a set of coupled differential equations for $u(t)$ and $v(t)$ is derived by

$$\dot{u}(t) = \frac{\mathrm{i}}{\hbar}\left[\frac{\kappa(t)}{2}u(t) - \eta(t)v(t)\right], \qquad \dot{v}(t) = \frac{\mathrm{i}}{\hbar}\left[\eta(t)u(t) - \frac{\kappa(t)}{2}v(t)\right]. \tag{21}$$

To solve these complex equations, it is more convenient to decompose u and v as $u = u_1 + \mathrm{i}u_2$ and $v = v_1 + \mathrm{i}v_2$, and introducing $x_{1,2} = u_1 \pm v_1$ and $x_{3,4} = u_2 \pm v_2$, we have

$$\begin{aligned} \dot{x}_1(t) &= -\frac{M(t)\Omega^2(t)}{m\omega_0}x_4(t), \qquad & \dot{x}_2(t) &= -\frac{m\omega_0}{M(t)}x_3(t), \\ \dot{x}_3(t) &= \frac{M(t)\Omega^2(t)}{m\omega_0}x_2(t), \qquad & \dot{x}_4(t) &= \frac{m\omega_0}{M(t)}x_1(t). \end{aligned} \tag{22}$$

Besides, defining the new variables $y_1 = x_4/x_1$ and $y_2 = x_3/x_2$, converting equation (22) into the nonlinear equations of $y_{1,2}$, and making the following Riccati transformations:

$$y_1(t) = -\frac{m\omega_0}{M(t)\Omega^2(t)}\frac{\mathrm{d}\ln Z_1(t)}{\mathrm{d}t}, \qquad y_2 = -\frac{M(t)}{m\omega_0}\frac{\mathrm{d}\ln Z_2(t)}{\mathrm{d}t}, \tag{23}$$

the second-order differential equations of the functions Z_j are then derived by

$$\ddot{Z}_1(t) - \left[\frac{\dot{M}(t)}{M(t)} + \frac{2\dot{\Omega}(t)}{\Omega(t)}\right]\dot{Z}_1(t) + \Omega^2(t)Z_1(t) = 0, \tag{24}$$

$$\ddot{Z}_2(t) + \frac{\dot{M}(t)}{M(t)}\dot{Z}_2(t) + \Omega^2(t)Z_2(t) = 0, \tag{25}$$

with the initial conditions $Z_j(0) = 1$ and $\dot{Z}_j(0) = 0$ (j=1,2). Finally, we find the solution of equation (21) in the form

$$\begin{aligned} u(t) &= \frac{1}{2}\left\{Z_1(t) + Z_2(t) + \mathrm{i}\left[\frac{1}{m\omega_0}\int_0^t \mathrm{d}t' M(t')\Omega^2(t')Z_2(t') + m\omega_0\int_0^t \mathrm{d}t'\frac{Z_1(t')}{M(t')}\right]\right\}, \\ v(t) &= \frac{1}{2}\left\{Z_1(t) - Z_2(t) + \mathrm{i}\left[\frac{1}{m\omega_0}\int_0^t \mathrm{d}t' M(t')\Omega^2(t')Z_2(t') - m\omega_0\int_0^t \mathrm{d}t'\frac{Z_1(t')}{M(t')}\right]\right\}. \end{aligned} \tag{26}$$

We are now in a position to determine the time evolution wavefunction. Assuming that the external magnetic field and the interaction turn on for a two-dimensional harmonic oscillator with constant mass and frequency at time $t = 0$, that is, starting from an initial two-mode particle number state $|n_1, n_2\rangle$, the time evolution wavefunction of the considered system at time t then becomes

$$\Psi_{n_1n_2}(q_1, q_2, t) = \sum_{m_1m_2;l_1l_2} \mathcal{R}_{m_1m_2;l_1l_2}(t)\mathcal{M}_{l_1l_2;n_1n_2}(t)\psi_{m_1m_2}(q_1, q_2), \tag{27}$$

where $\psi_{m_1m_2}(q_1, q_2) = \prod_j^2 \varphi_{m_j}(q_j)$ with $\varphi_{m_j}(q_j) = \langle q_j|m_j\rangle = [\xi/(2^{m_j} m_j!\pi^{1/2})]^{1/2} (-1)^{m_j}\xi^{-m_j} \exp\left(\xi^2q_j^2/2\right)d^{m_j} \exp\left(-\xi^2q_j^2\right)/\left(dq_j^{m_j}\right)$ being the eigenfunctions of a simple harmonic oscillator in which $\xi \equiv m\omega_0/\hbar$.

The matrix element of the rotation operator $R_z(t)$ is now given by

$$\begin{aligned}\mathcal{R}_{m_1m_2;l_1l_2}(t) &= \langle m_1, m_2|\exp[-\xi_c(t)(J_+ - J_-)]|l_1, l_2\rangle \\ &= \langle m_1, m_2|\exp[-\tan\xi_c(t)J_+]\exp[-2\ln\cos\xi_c(t)J_0]\exp[\tan\xi_c(t)J_-]|l_1, l_2\rangle \\ &= \left[\frac{1-\sin\xi_c(t)}{\cos\xi_c(t)}\right]^{l_1}\left[\frac{\cos^2\xi_c(t)+\sin\xi_c(t)}{\cos\xi_c(t)}\right]^{l_2}\delta_{m_1l_1}\delta_{m_2l_2},\end{aligned} \tag{28}$$

where

$$J_+ = a_1^\dagger a_2, \qquad J_- = a_1a_2^\dagger, \qquad J_0 = \tfrac{1}{2}\left(a_1^\dagger a_1 - a_2^\dagger a_2\right) \tag{29}$$

are the generators of the $su(2)$ Lie algebra realized by two-mode boson operators $a_1(a_1^\dagger)$ and $a_2\left(a_2^\dagger\right)$, and they satisfy the closed commutation relations $[J_+, J_-] = 2J_0$, $[J_0, J_\pm] = \pm J_\pm$. Here, from the second step to the third in equation (28) we have used the Baker–Campbell–Hausdorff decomposition formula [14].

By analogy to the calculation procedure for matrix elements of the squeezing operator in quantum optics [15], for the evolution operator $\mathcal{U}_1(t)\mathcal{U}_2(t)$, using equation (18) we have

$$\mathcal{M}_{l_1l_2;n_1n_2}(t) = \langle l_1|\mathcal{U}_1(t)|n_1\rangle\langle l_2|\mathcal{U}_2(t)|n_2\rangle$$

$$= \begin{cases} \dfrac{(-1)^{(l_1+l_2)/2}}{\left(\frac{l_1}{2}\right)!\left(\frac{n_1}{2}\right)!\left(\frac{l_2}{2}\right)!\left(\frac{n_2}{2}\right)!}\dfrac{(l_1!n_1!l_2!n_2!)^{1/2}}{u}\left(\dfrac{v}{2u}\right)^{(l_1+l_2)/2}\left(\dfrac{v^*}{2u}\right)^{(n_1+n_2)/2} \\ \quad\times F\left(-\dfrac{l_1}{2}, -\dfrac{n_1}{2}, \dfrac{1}{2}, -\dfrac{1}{|v|^2}\right)F\left(-\dfrac{l_2}{2}, -\dfrac{n_2}{2}, \dfrac{1}{2}, -\dfrac{1}{|v|^2}\right), \\ \qquad\qquad \text{for} \quad l_1, n_1 \quad \text{and} \quad l_2, n_2 \quad \text{even}, \\ \dfrac{(-1)^{(l_1+l_2)/2-1}}{\left(\frac{l_1-1}{2}\right)!\left(\frac{n_1-1}{2}\right)!\left(\frac{l_2-1}{2}\right)!\left(\frac{n_2-1}{2}\right)!}\dfrac{(l_1!n_1!l_2!n_2!)^{1/2}}{u^3}\left(\dfrac{v}{2u}\right)^{(l_1+l_2)/2-1}\left(\dfrac{v^*}{2u}\right)^{(n_1+n_2)/2-1} \\ \quad\times F\left(-\dfrac{l_1-1}{2}, -\dfrac{n_1-1}{2}, \dfrac{3}{2}, -\dfrac{1}{|v|^2}\right)F\left(-\dfrac{l_2-1}{2}, -\dfrac{n_2-1}{2}, \dfrac{3}{2}, -\dfrac{1}{|v|^2}\right), \\ \qquad\qquad \text{for} \quad l_1, n_1 \quad \text{and} \quad l_2, n_2 \quad \text{odd}, \\ 0, \qquad \text{otherwise}, \end{cases} \tag{30}$$

where $F(a, b, c, x)$ is the hypergeometric function.

It is seen that the evolution wavefunction is far from a trivial form even if in the special case. For example, for the two-dimensional harmonic oscillator with a constant mass and frequency in a static magnetic field we have $M(t) \equiv m$, $\omega(t) \equiv \omega_0$, $\omega_c(t) = eB_0/(mc) \equiv \omega_{c0}$, and $\Omega(t) = \left(\omega_0^2 + \omega_{c0}^2/4\right)^{1/2} \equiv \Omega_0$. It turns out to be $\xi_c = \omega_{c0}t/2$, $Z_1 = Z_2 = \cos\Omega_0 t$, and

$$\begin{aligned} u(t) &= \cos\Omega_0 t + \frac{\mathrm{i}(\Omega_0^2 + \omega_0^2)}{2\Omega_0\omega_0}\sin\Omega_0 t, \\ v(t) &= \frac{\mathrm{i}\omega_{c0}^2}{8\Omega_0\omega_0}\sin\Omega_0 t, \end{aligned} \tag{31}$$

from which it follows that

$$\Psi_{n_1n_2}(q_1, q_2, t) = \sum_{m_1,m_2,r,s} \frac{(m_1!m_2!n_1!n_2!)^{1/2}}{r!s!(n_1-2r)!(n_2-2s)![(m_1-n_1)/2+r]![(m_2-n_2)/2+s]!}$$
$$\times \left[\frac{1-\sin(\omega_{c0}t/2)}{\cos(\omega_{c0}t/2)}\right]^{m_1} \left[\frac{\cos^2(\omega_{c0}t/2)+\sin(\omega_{c0}t/2)}{\cos(\omega_{c0}t/2)}\right]^{m_2}$$
$$\times \left[-\frac{\omega_{c0}^4 \sin^2\Omega_0 t}{256\Omega_0^2\omega_0^2}\right]^{r+s} \left[-\mathrm{i}\frac{\omega_{c0}^2 \sin\Omega_0 t}{16\Omega_0\omega_0}\right]^{(m_1+m_2-n_1-n_2)/2}$$
$$\times \left[\cos\Omega_0 t + \mathrm{i}\frac{(\Omega_0^2+\omega_0^2)\sin\Omega_0 t}{2\Omega_0\omega_0}\right]^{-(m_1+m_2+n_1+n_2+2)/2} \psi_{m_1m_2}(q_1, q_2). \quad (32)$$

If the magnetic field vanishes, we then obtain the stationary state wavefunction $\Psi_{n_1n_2}(q_1, q_2, t) = \exp[-\mathrm{i}(n_1 + n_2 + 1)\omega_0 t]\psi_{n_1n_2}(q_1, q_2)$ of the two decoupled harmonic oscillators.

In conclusion, we have obtained the exact analytical expressions of the time evolution operator and the evolution wavefunction of the two-dimensional harmonic oscillator with time-dependent mass and frequency in a static magnetic field using the alternative operator-algebraic method. In contrast with the usual Lie algebraic approach [5, 6, 11, 12], the two major advantages of this method are that, first, there is no need to solve many sets of parameter equations and, second, the time evolution operator and evolution wavefunction can be formulated analytically in terms of the transformation coefficients $u(t)$ and $v(t)$. Once the standard second-order differential equations (24) and (25) are solved and the integrals in equation (26) are carried out, the explicit expressions of the time evolution wavefunction can be straightforwardly written.

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